The Three-Nucleon System Near the N-d Threshold

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Abstract

The three-nucleon system is studied at energies a few hundred keV above the N-d threshold. Measurements of the tensor analyzing powers T_{20} and T_{21} for p-d elastic scattering at $E_{c.m.} = 432$ keV are presented together with the corresponding theoretical predictions. The calculations are extended to very low energies since they are useful for extracting the p-d scattering lengths from the experimental data. The interaction considered here is the Argonne V18 potential plus the Urbana three-nucleon potential. The calculation of the asymptotic D- to S-state ratio for ^{3}H and ^{3}He, for which recent experimental results are available, is also presented.

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Study of the three-nucleon system at low energies provides a stringent test of our understanding of the nuclear dynamics and the nucleon-nucleon (NN) interaction. In Ref. [1], detailed comparisons have been performed between experimental data and the corresponding theoretical predictions for N-d scattering for $0.6 \leq E_{c.m.} \leq 2.0$ MeV. Good agreement was observed for the cross section and the tensor analyzing powers T_{20}, T_{21}, and T_{22}, but significant differences were found in the vector analyzing power A_y for N-d scattering and the analyzing power iT_{11} for p-d scattering. It is also important to experimentally test the theoretical calculations at lower energies. One motivation is to extract the p-d scattering lengths, for which past experimental results [2] have persistently disagreed with theoretical estimates [3]. A further advantage is that at low energies the contributions from higher partial waves are reduced, so that the remaining important partial waves can be better investigated. Another motivation for these comparisons is to test the theoretical p-d scattering wavefunctions which have recently been used to calculate the astrophysical S-factor and analyzing powers for the $^2\mathrm{H}(p,\gamma)^3\mathrm{He}$ reaction at very low energies [4].

Below $E_{c.m.} = 0.6$ MeV, the cross section is mainly governed by S-wave scattering. As a result the polarization observables, which are influenced by other partial waves, are small and difficult to determine experimentally. The present paper reports measurements of T_{20} and T_{21} for p-d elastic scattering at $E_{c.m.} = 432$ keV. These data are compared to calculations utilizing the Pair-Correlated Hyperspherical Harmonic (PHH) basis [5] to construct the scattering wave function. The corresponding reactance matrix ($\mathcal{R}$ matrix) is obtained by means of the Kohn variational principle, as described in Ref. [6]. The calculations have been done using the AV18 [7] potential plus the three-nucleon interaction (TNI) of Urbana (UR) [8]. They have been extended to lower energies in order to investigate the behavior of the $\mathcal{R}$-matrix elements near zero energy, and to evaluate the p-d scattering lengths $^2a_{pd}$ and $^4a_{pd}$.

The asymptotic S- and D-state constants for which recent accurate results are available [9–11] have also been calculated. These quantities involve the interaction between three nucleons with total angular momentum $J^\pi = \frac{1}{2}^+$, which is a very important partial wave in low-energy N-d scattering. The properties of the three-nucleon bound states, such as binding energy and asymptotic constants, and low-energy elastic scattering observables are closely related. A correct theoretical description of the three-nucleon system should be able to reproduce both types of data.

The measurements were carried out using tensor-polarized deuteron beams from the atomic beam polarized ion source [12] at the Triangle Universities Nuclear Laboratory (TUNL). The beams were accelerated to $E_d = 1.3$ MeV using the FN tandem accelerator, and then directed into a 62-cm diameter scattering chamber. Thin hydrogenated carbon targets were used [13] which consisted of approximately 1×10^{18} and 2×10^{18} hydrogen and carbon atoms/cm^2, respectively. The deuteron beam loses ≈ 5 keV in these targets, leading to an average energy of $E_{c.m.} = 432 \pm 3$ keV, where the error in-

cludes the uncertainty in the incident energy. The use of thin targets is very important at low energies for minimizing energy loss and straggling effects.

The beam polarization was determined to $\pm 3\%$ using the ^{3}He(d,p) reaction in an online polarimeter located behind the scattering chamber. The polarimeter is described in Ref. [14]; the calibration has been extended down to $E_d = 1.3$ MeV. This reaction is excellent for deuteron tensor polarimetry at low energies due to the large cross section and tensor analyzing powers. The data were taken using three spin states with tensor polarizations $p_{ZZ} \approx \pm 0.7$ and $p_{ZZ} \approx 0$. The spin states were cycled approximately once every second, in order to minimize the effects of slow changes in beam position, target thickness, or amplifier gain. Tests for false asymmetries were carried out by measuring ^{197}Au(d,d) scattering at $\theta_{lab} = 40°$ under identical conditions as the p-d measurements. For $E_d = 1.3$ MeV, all of analyzing powers for ^{197}Au(d,d) are expected to be $< 10^{-4}$ [15]. The results were consistent with zero at the level of 5×10^{-4}, the statistical uncertainty of the measurement.

Scattered deuterons and protons were detected with silicon surface barrier detectors located between 15 and 25 cm from the target. At certain angles 2- or 5-μm mylar foils were placed in front of the detectors to stop heavy nuclei recoiling from the target, or to separate the deuterons from p(d,d) scattering from protons due to the ^{12}C(d,p$_1$) reaction. Backgrounds were typically less than 3%, and were subtracted using linear or exponential fits to background regions on either side of the peak of interest. The final errors include the uncertainty in the background subtraction arising from the background region fitted, the functional form used form the background, and the analyzing power of the background.

Angular distributions of T_{20} and T_{21} are shown in Fig. 1. The error bars include contributions from counting statistics, background subtraction, and dead time corrections, but not the uncertainties in absolute beam polarization.

The theoretical methods developed in Ref. [6] can be applied to n-d as well as p-d scattering below the breakup threshold. When realistic NN potentials and TNI terms are considered, the corresponding results allow for meaningful detailed comparisons with experimental data. Scattering waves up to orbital angular momentum $L = 3$ have been taken into consideration in the construction of the scattering wave function. The calculated T_{20} and T_{21} are shown in Fig. 1, and are seen to be in remarkably good agreement with the experimental measurements. A particularly stringent test of the theoretical approach is provided by the T_{20} data, whose complicated structure is due to interference effects between the nuclear and Coulomb interactions. The magnitude of the polarization observables gives a measure of the importance of $L > 0$ partial waves, for if only S-wave scattering were considered, all of the polarization observables would be zero.

The calculations have been extended to lower energies in order to make predictions for the effective range functions and N-d scattering lengths. In the past the doublet scattering length in N-d scattering has been calculated for a variety of interaction models [16]. A correct approach to the problem

requires that the interaction model considered reproduces the three-nucleon binding energy. Apart from small differences between the various models, a reasonable description is obtained for the n-d system, in particular when the AV18+UR interaction is adopted. The calculated value ${}^2a_{nd} = 0.63$ fm [17] compares well with the experimental value of 0.65 ± 0.04 fm [18]. For the p-d case, all the calculations agree with the value ${}^2a_{pd} \approx 0$. Extraction of the experimental value is complicated by the large curvature of the effective range function when the energy approaches zero.

An effort is currently underway at TUNL to measure the p-d differential cross section at $E_{c.m.} \approx 170$ keV and $E_{c.m.} \approx 210$ keV [13]. For $E_{c.m.} < 300$ keV the cross section is dominated by Coulomb scattering and it is justified to consider the nuclear phase shift in only the S- and P-waves. The $\mathcal{R}$ matrices for $J^\pi = \frac{1}{2}^+$ and $\frac{3}{2}^+$ are then scalars. Following Ref. [3], an effective range expansion for the corresponding element ${}^J R_{00} = \tan\delta_0^J$ can be performed. To be explicit, let us concentrate on the S-wave $J^\pi = \frac{1}{2}^+$ phase shift. For p-d scattering the effective range function is

$$K(E) = C_0^2(\eta) k \cot\delta_0(k) + 2k\eta h(\eta), \tag{1}$$

where $\eta = 2Me^2/3\hbar^2 k$ is the Coulomb parameter, M the nucleon mass, $C_0^2 = 2\pi\eta/(\mathrm{e}^{2\pi\eta} - 1)$, $h(\eta) = -ln(\eta) + \mathrm{Re}\psi(1 + i\eta)$, and ψ is the digamma function. Again, the calculations have been done using the AV18+UR nuclear potential model and the Coulomb interaction. Other electromagnetic terms of the AV18 potential (such as the vacuum polarization and magnetic moment terms) have been neglected in order to simplify the calculations, since the asymptotic solutions can then be expressed in terms of the Coulomb functions. The inclusion of those neglected terms produces only small changes in the scattering lengths [19].

The phase shift $\delta_0^{1/2}$ has been calculated for several energies with $E_{c.m.} <$ 450 keV. The calculated values of the doublet effective range function are plotted in Fig. 2, where the arrows indicate the energies at which the experiments were performed. These numerical results reveal a pole in the effective range function near zero energy. A good fit to the numerical results can be obtained assuming

$$K(E) = \frac{-1/\,{}^2a_{pd} + \beta E}{1 + E/E_0} \ . \tag{2}$$

This singular behavior was anticipated in Ref. [3] where S-wave potentials were used and it is corroborated here for realistic nuclear interactions. The fitted values obtained for the free parameters are a doublet scattering length of ${}^2a_{pd} = 0.024$ fm, $\beta = -0.076$ fm^{-1} keV^{-1} and $E_0 = 3.13$ keV corresponding to the solid curve in Fig. 2. A direct calculation for this quantity at zero energy gives ${}^2a_{pd} = 0.027$ fm, in good agreement with the extrapolated result. When all the electromagnetic terms of the AV18 potential are properly taken into account the value ${}^2a_{pd} = -0.022$ fm is obtained [17]. Since experiments for the p-d system below $E_{c.m.} = 150$ keV are extremely difficult, the form given in

Eq. (2) is a useful guide for an energy-dependent PSA intended to determine the p-d scattering lengths.

The problems with extrapolation to $E = 0$ are not present in the $J^\pi = \frac{3}{2}^+$ S-wave state. The corresponding effective range function has a smooth behavior near the N-d threshold [3]. The quartet scattering lengths, calculated using the AV18+UR model, are ${}^4a_{nd}$=6.33 fm and ${}^4a_{pd}$=13.8 fm. Here again, for the n-d system the theoretical estimate is quite close to the experimental value ${}^4a_{nd} = 6.35 \pm 0.02$ fm [18].

Further information on the structure of the three-nucleon system can be obtained through the study of the asymptotic constants. It is well known that the $J^\pi = \frac{1}{2}^+$ elastic scattering amplitude and the properties of the three-nucleon bound states are closely related. It is therefore of interest to compare the predictions of the AV18+UR potential model to recent experimental results [9–11]. For the absolute values of the S-state asymptotic constant C_S, the D-state asymptotic constant C_D and the ratio $\eta = C_D/C_S$ we obtain: $C_S = 1.854$, $C_D = 0.0798$, $\eta = 0.0430$ for ^{3}H, and $C_S = 1.878$, $C_D = 0.0752$, $\eta = 0.0400$ for ^{3}He. From the above results it can be seen that the ratio η is lower than previous calculations [20] and agree well with the experimental results: $\eta({}^3\mathrm{H}) = 0.0411 \pm 0.0013 \pm 0.0012$ [9] and 0.0431 ± 0.0025 [11], $\eta({}^3\mathrm{He}) = 0.0386 \pm 0.0045 \pm 0.0012$ [10]. Since η gives a measure of the D-state amplitude, these results are expected, to some extent, due to a weaker tensor component present in the AV18 NN potential.

The study of the three-nucleon properties near the N-d threshold was the aim of the present paper. The theoretical calculations agree well in magnitude and shape with the highly-accurate T_{20} and T_{21} data at 432 keV where Coulomb effects are of considerable importance. These results increase our confidence in the theoretical p-d scattering wavefunctions used recently to calculate the S-factor and analyzing powers of the ${}^2\mathrm{H}(p,\gamma)$ reaction at very low energies [4].

The calculated $J^\pi = \frac{1}{2}^+$ S-wave effective range function for p-d scattering reveals a pole a few keV above the N-d threshold and predicts a doublet scattering length ${}^2a_{pd} = 0.024$ fm. Both values are consistent with recent Faddeev calculations. The calculated effective range function can serve as a guide for future experiments intended to determine the a_{pd} scattering lengths.

The calculated S- and D-state asymptotic normalization constants for ^{3}He and ^{3}H are in excellent agreement with recent measurements.

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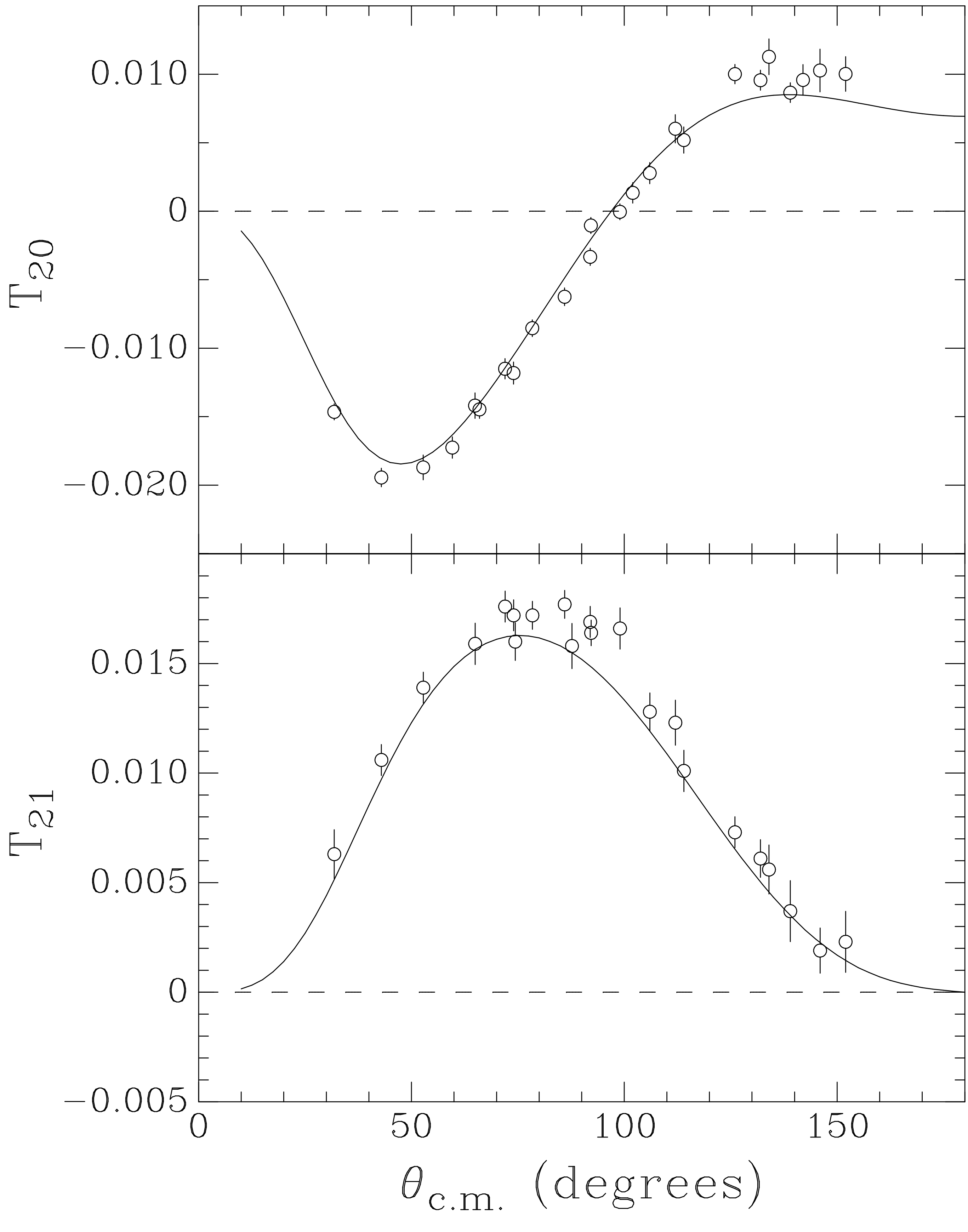
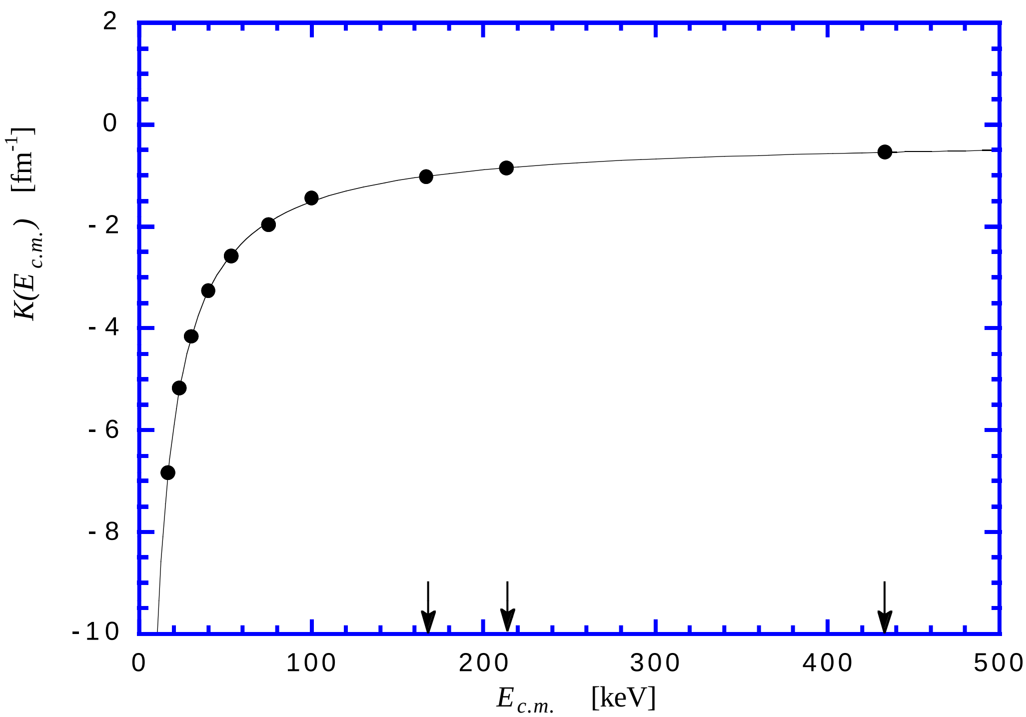
FIGURES

FIG. 1. The tensor analyzing powers T_{20} and T_{21} for p(d,d) scattering at $E_{c.m.} = 432$ keV, as a function of center-of-mass angle. The experimental data are shown as circles and the solid curves show the theoretical calculations.

FIG. 2. Effective range function values (solid circles) calculated with the p-d doublet phase shifts at various energies. The solid curve is the fit obtained with Eq. (2). The arrows indicate the energies at which experiments have been performed at TUNL.



0.010
0
−0.010
−0.020
T_{20}
0.015
0.010
0.005
0
−0.005
T_{21}
0
50
100
150
$\theta_{c.m.}$ (degrees)



2
0
- 2
- 4
- 6
- 8
- 1 0
0
1 0 0
2 0 0
3 0 0
4 0 0
5 0 0
$K(E_{c.m.})$ [fm^{-1}]
$E_{c.m.}$ [keV]